

AA-114

April-2018

Third Year B.B.A., Sem.-VI**Five Years Integrated MBA Course
(Operations Research)****Time : 3 Hours]****[Max. Marks : 100**

- Instructions :** (1) Graph papers shall be provided on demand.
(2) Non-programmable scientific calculators are allowed.

1. Attempt any **two** :**20**

- (i) Explain the assumptions of a linear program. What are its limitations ?
- (ii) A company manufactures three products P_1 , P_2 and P_3 . The machine hour requirements and labour hour requirements to process the three products and the maximum available machine hours per week for each machine type and maximum available labour hours per week are summarized in the table. The selling price per unit and product cost per unit are also summarized in the same table. The company wants to limit the production volume per week of the product P_3 to utmost 35 units. Formulate (do not solve) a linear programming model to find the production volume per week of each product such that the total profit is maximized.

Machine	Product			Maximum available hours per week
	Machine hours required			
	P ₁	P ₂	P ₃	
1	4	6	3	500
2	3	—	2	300
3	5	7	8	600
Labour	8	2	4	200

Selling per unit (₹)	500	400	550
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Product cost per unit (₹)	350	280	390
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(iii) Solve the following linear programming problem using graphical method.

$$\text{Maximize } Z = 100x_1 + 80x_2$$

Subject to constraints

$$5x_1 + 10x_2 \leq 50$$

$$8x_1 + 2x_2 \geq 16$$

$$3x_1 - 2x_2 \geq 6$$

$$x_1, x_2 \geq 0$$

2. Attempt any **two** :

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(i) Use the simplex method to solve the following linear programming problem.

$$\text{Maximize } Z = 10x_1 + 15x_2 + 20x_3$$

Subject to constraints

$$2x_1 + 4x_2 + 6x_3 \leq 24$$

$$3x_1 + 9x_2 + 6x_3 \leq 30$$

$$x_1, x_2, x_3 \geq 0$$

(ii) Solve the following linear programming problem using Big M method.

$$\text{Minimize } Z = 10x_1 + 15x_2 + 20x_3$$

Subject to constraints

$$2x_1 + 4x_2 + 6x_3 \geq 24$$

$$3x_1 + 9x_2 + 6x_3 \geq 30$$

$$x_1, x_2, x_3 \geq 0$$

(iii) Solve the following linear programming problem using dual simplex method.

$$\text{Minimize } Z = x_1 + x_2$$

Subject to constraints

$$2x_1 + x_2 \geq 2$$

$$-x_1 - x_2 \geq 1$$

$$x_1, x_2 \geq 0$$

3. (i) Write the dual of the following linear programming problem

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$$\text{Maximize } z = 3x_1 + 2x_2 - 5x_3$$

Subject to constraints

$$x_1 + x_2 \leq 2$$

$$2x_1 + x_2 + 6x_3 \leq 6$$

$$x_1 - x_2 + 3x_3 = 0$$

$$x_1, x_2 \geq 0 \text{ and } x_3 \text{ is unrestricted in sign.}$$

(ii) Giapetto's Woodcarving, Inc., manufactures two types of wooden toys : Soldiers and trains. A soldier sells for ₹ 27 and uses ₹ 10 worth of raw materials. Each soldier that is manufactured increases Giapetto's variable labour and overhead costs by ₹ 14. A train sells for ₹ 21 and uses ₹ 9 worth of raw materials. Each train built increases Giapetto's variable labour and overhead costs by ₹ 10. The manufacture of wooden soldiers and trains requires two types of skilled labour : carpentry and finishing. A soldier requires 2 hours of finishing labour and 1 hour of carpentry labour. A train requires 1 hour of finishing and 1 hour of carpentry labour. Each week, Giapetto can obtain all the needed raw material, but only 100 finishing hours and 80 carpentry hours. Demand for trains is unlimited, but at most 40 soldiers are bought each week. Giapetto wants to maximize weekly profit. To maximize Giapetto's weekly profit, Giapetto's situation is formulated as a Mathematical model and solved for optimal solution as given in the table.

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Basis	x_1	x_2	s_1	s_2	s_3	b_i
3 x_1	1	0	1	-1	0	20
2 x_2	0	1	-1	2	0	60
0 s_3	0	0	-1	1	1	20
c_j	3	2	0	0	0	
z_j	3	2	1	1	0	
$c_j - z_j$	0	0	-1	-1	0	

Use this optimal table to answer the following questions.

- (a) If soldiers contribute ₹ 3.50 to profit, what would be the effect on the optimal solution to the Giapetto problem ?
- (b) If 90 finishing hours are available instead of 100, what would be the effect on the optimal solution to the Giapetto problem ?
- (c) What should be the demand for soldiers so that the current basis remains optimal ?
- (d) Giapetto is considering manufacturing toy boats. A toy boat uses 2 carpentry hours and 1 finishing hour. Demand for toy boats is unlimited. If a toy boat contributes ₹ 3.5 to profit, should Giapetto manufacture any toy boats ?

4. (i) The following table shows all the necessary information on the availability of supply to each warehouse, the requirement of each market and unit transportation cost (in ₹) from each warehouse to each market.

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Warehouse	Market				Supply
	P	Q	R	S	
A	6	3	5	4	22
B	5	9	2	7	15
C	5	7	8	6	8
Demand	7	12	17	9	

The shipping clerk has worked out the following schedule from experience : 12 units from A to Q, 1 unit from A to R, 9 units from A to S, 15 units from B to R, 7 units from C to P and 1 unit from C to R.

- (a) Check and see if the clerk has the optimal schedule.
- (b) Find the optimal schedule and minimum total transportation cost.
- (c) If the clerk is approached by a carrier of route C to Q, who offers to reduce his rate in the hope of getting some business, by how much the rate should be reduced before the clerk, will offer him the business ?

- (ii) A company has factories at F_1 , F_2 and F_3 which supply to warehouses at W_1 , W_2 and W_3 . Weekly factory capacities use 200, 160 and 90 units respectively. Weekly warehouse requirement are 180, 120 and 150 units, respectively. Unit shipping costs (in rupees) are as follows :

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		Warehouse			Supply
		W_1	W_2	W_3	
Factory	F_1	16	20	12	200
	F_2	14	8	18	160
	F_3	26	24	16	90
Demand		180	120	150	

Determine the optimal distribution for this company to minimize total shipping cost. Are there multiple optimal solutions ? If yes, identify them.

OR

General Ford has two plants, two warehouses, and three customers. The locations of these are as follows :

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Plants : Detroit and Atlanta

Warehouses : Denver and New York

Customers : Los Angeles, Chicago and Philadelphia

Cars are produced at plants, then shipped to customers. Detroit can produce 150 cars per week and Atlanta can produce 100 cars per week. Los Angeles require 80 cars per week; Chicago, 70; and Philadelphia, 60. It costs \$ 10,000 to produce a car at each plant, and the cost of shipping a car between two cities is given in the table. Determine how to meet General Ford's weekly demands at minimum cost.

		To (\$)	
		Denver	New York
From	Detroit	1253	637
	Atlanta	1398	841

		To (\$)		
		Los Angeles	Chicago	Philadelphia
From	Denver	1059	996	1691
	New York	2786	802	100

- (i) A college is having a degree programme for which the effective semester time available is very less and the programme requires field work. Hence a few hours can be saved from the total number of class hours and can be utilized for the field work. Based on past experience, the college has estimated the number of hours required to teach each subject by each faculty. The course in its present semester has 5 subjects and the college has considered 6 existing faculty members to teach these courses. The objective is to assign the best 5 teachers out of these 6 faculty members to teach 5 different subjects so that the total number of class hours required is minimized. The data of the problem is summarized in the table. Solve this assignment problem optimally.

		Subject				
		S ₁	S ₂	S ₃	S ₄	S ₅
Faculty	F ₁	30	39	31	38	40
	F ₂	43	37	32	35	38
	F ₃	34	41	33	41	34
	F ₄	39	36	43	32	36
	F ₅	32	49	35	40	37
	F ₆	36	42	35	44	42

- (ii) The flight timings between two cities X and Y, are as given in the following tables. The minimum layover time of any crew in either of the cities is 2 hours. Determine the base city for each crew so that the sum of the layover times of all the crew members in non-base cities is minimized.

Flight Timings from City X to City Y

Flight Number	Departure time (from City X)	Arrival time (to City Y)
101	5 am	6 am
102	9 am	10 am
103	1 pm	2 pm
104	6 pm	7 pm

Flight Timings from City Y to City X

Flight Number	Departure time (from City Y)	Arrival time (from City X)
201	6 am	7 am
202	9 am	10 am
203	3 pm	4 pm
204	10 pm	11 pm

- (iii) A salesman wants to visit cities 1, 2, 3 and 4. He does not want to visit any city twice before completing the tour of all the cities and wishes to return to his home city, the starting station. Cost of going from one city to another in rupees is given in the table. Find the least cost route.

		To city			
		1	2	3	4
From City	1	–	3	8	5
	2	4	–	14	3
	3	4	5	–	2
	4	7	8	13	–

